CORRECTION EXAMEN JANVIER 2023 MAT 103

Exercice 1

1) $\mathcal{D}_f = \mathbb{R}$. Puisque $\forall x \in \mathbb{R}, x^2+3 \geq 0$, $\mathcal{D}_g = \mathbb{R}$

$\forall x \in \mathbb{R}, x^2+2x+1 = (x+1)^2 \geq 0$ et $=0$ ssi $x=-1$.

Donc $\mathcal{D}_h = \mathbb{R} \setminus \{-1\}$.

$$f'(x) = (2x+3x^2)e^{3x+1}$$

$$g'(x) = \frac{12x}{2\sqrt{x^2+3}} = \frac{6x}{\sqrt{x^2+3}}$$

2) $\frac{\partial f}{\partial x}(x,y) = 2xy+5y \quad \frac{\partial f}{\partial y} = x^2+5x$

Exercice 2 $1+x^2>0$ Donc $\mathcal{D}_f = \mathbb{R}$

(1) $F(x) = \frac{3}{2}\ln|1+x^2| = \frac{3}{2}\ln(1+x^2)$ $(F' = f)$

$(1-x)^2 = 0 \iff x=1$ donc $\mathcal{D}_g = \mathbb{R} \setminus \{1\}$.

$G(x) = \frac{1}{1-x}$ car $G' = g$

$H(x) = \frac{1}{2}e^{x^2+2x}$

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(2) $\int_0^1 t^2-2t-3\,dt = \left[\frac{t^3}{3}-t^2-3t\right]_0^1 = \frac{1}{3}-4 = \frac{-11}{3}$

$\int_0^1 \frac{e^t}{e^t+1}\,dt = \left[\ln|e^t+1|\right]_0^1 = \ln(e+1)-\ln 2$
$= \ln\left(\frac{1+e}{2}\right).$

Exercice 3 $A \in M_{22}$ $B \in M_{23}$ AB possible
BA non car $\underset{\text{col } B}{3} \neq \underset{\text{ligne } A}{2}$.

(1)

$AB = \begin{pmatrix} 3 & -2 & -4 \\ 5 & -4 & -5 \end{pmatrix}$

(2) $\det A = 4-3 = 1 \neq 0$. $A^{-1} = \frac{1}{1}\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$.

(3) En faisant AA^{-1} et vérifier que c'est I_2.

(4) $A(4I-A) = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} \overset{2}{\overbrace{4-2}} & -1 \\ -3 & \underset{2}{\underbrace{4-2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$

D'où $A^{-1} = 4I - A$

Exercice 4 $u_0 = 2$ $u_{n+1} = 3u_n - 2$.

$u_1 = 4$ $u_2 = 10$. $\begin{cases} u_1 - u_0 = 2 \\ u_2 - u_1 = 6 \end{cases} \Rightarrow (u_n)_n$ non arith.

$\begin{cases} \frac{u_1}{u_0} = 2 \\ \frac{u_2}{u_1} = \frac{10}{4} \neq 2 \end{cases} \Rightarrow (u_n)$ non géom.

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$v_{n+1} = 3u_n - 2 - 1 = 3 v_n.$

2a) D'où $v_n = v_0 3^n = 3^n$

$u_n = 1 + 3^n.$

2b) $3 \geq 1$ donc (u_n) croissante (strict^t) ou car $\left(u_{n+1} - u_n = 3^{n+1} - 3^n = 2 \, 3^n > 0 \right)$

~~elle est stri~~ $u_1 > u_0$ donc (u_n) n'est pas décroissante.

(3) $u_n \geq 1000 \iff 3^n \geq 999 \underset{\ln \text{ strict}^t \nearrow}{\iff} n \ln 3 \geq \ln 999$

$\underset{\ln 3 > 0 \text{ car } 3 > 1}{\iff} n \geq \dfrac{\ln 999}{\ln 3} \simeq 6,28$

$N = 7$ convient.